An age structured demographic model of technology

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Abstract

At the heart of technology transitions lie complex processes of technology choices. Understanding and planning sustainability transitions requires modelling work, which necessitates a theory of technology substitution. A theoretical model of technological change and turnover is presented, intended as a methodological paradigm shift from widely used conventional modelling approaches such as cost optimisation. It follows the tradition of evolutionary economics and evolutionary game theory, using ecological population growth dynamics to represent the evolution of technology populations in the marketplace, with substitutions taking place at the level of the decision-maker. Extended to use principles of human demography or the age structured evolution of species in interacting ecosystems, this theory is built from first principles, and through an appropriate approximation, reduces to a form identical to empirical models of technology diffusion common in the technology transitions literature. Using an age structure, it provides the appropriate groundwork and theoretical framework to understand interacting technologies, their birth, ageing and mutual substitution. This analysis provides insight in explaining the nature and origin of observed timescales of technology transitions, in terms of technology life expectancies, the dynamic process of production capacity expansion or collapse and its timescales, in what is termed a demographic phase. While this model contributes to the general understanding of technological change, the information in this work is intended to be used practically for the parameterisation of technology diffusion in large scale models of technology systems when measured data is unknown or uncertain, as is the case for new technologies, notably for modelling future energy systems and greenhouse gas emissions.

Keywords: Technology diffusion, Technology transitions, Socio-technical regimes, Evolutionary game theory, Evolutionary economics

1. Introduction

1.1. Creative Destruction

Systems of technologies and their interactions are notoriously complex to model and understand, but such an understanding is crucial for anticipating and informing the planning of sustainability transitions. Socio-technical systems generate crucial *societal functions* (Geels, 2002, 2005), and these services and their demand are in a continuous evolution. Meanwhile, the evolution of technology generates new opportunities to society that enable the creation of activities that did not exist previously, producing a complex interaction between technology, society and the economy, even generating economic growth through Schumpeter's widely discussed but not well understood process of 'creative destruction' (Schumpeter 1934; see also for instance Nelson and Winter 1982). Technological change occurs through a gradual process of technology substitutions which stems from a continuous stream of decision-making performed by a myriad of actors involved in the operation of technology or the consumption of the services it generates (Grübler, 1998, Grübler et al., 1999). This spans from, for example, the power sector, vehicles for transport, communications and information technologies, heating, cooling and lighting equipment and so on, in other words, in sectors performing societal functions, of which the underlying generation technologies, and their associated socio-technical

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standards, are not unique. Change in such sectors occurs through the choices of consumers or investors facing various alternatives and incomplete information, and these decisions are based, in a context of *bounded rationality*, on diverse sets of considerations and constraints (concepts well explored by Nelson and Winter, 1982).

The process of technological change is not currently well described by any generally accepted theory (observing the contrast between approaches for instance of Geels, 2002, Grübler, 1998, Safarzynska and van den Bergh, 2010). However, a significant and well known body of empirical literature exists that consistently describes the process of technology substitutions through gradual S-shaped curves (e.g. Mansfield 1961; the best reviews probably remain the work of Grübler 1998, Grübler et al. 1999). These trends strongly suggest a parallel with ecological theories of population dynamics (e.g. Metcalfe, 2004), the best known and most appropriate consisting in using the Lotka-Volterra system of population growth equations for the competition of species (for an explanation and history see Andersen, 1994), or the replicator dynamics equation of evolutionary game theory (Hofbauer and Sigmund, 1998). While this idea has strong support in the fields of evolutionary and industrial economics, it also makes intuitive sense to perceive competing technologies (or even socio-technical systems) in the marketplace similarly to competing species in ecosystems (or even competing sub-ecosystems and food chains). The parallel has been brought much further with the development of evolutionary game theory (Hodgson and Huang 2012; see also Hofbauer and Sigmund 1998), the key authors of which, such as Maynard Smith and Price, were acutely aware of the strong analogy that could be drawn between the mathematics of the evolution of genotype frequencies and their selection in a population in biology, and the process of innovation and technology diffusion in economics. In addition to providing a definition to the concept of bounded rationality, this strand of literature demonstrates that the parallel, although described with yet insufficient precision, is more than just intuitive (Metcalfe, 2008, 2004).

The description of technological change or technology evolution following parallels with ecology currently remains in the conceptual and theoretical domain (for a review, see Safarzynska and van den Bergh, 2010) or in stylised form (e.g. 'history-friendly models' of Malerba et al. 1999; or Safarzynska and van den Bergh 2012) and not quite adapted to actual quantitative applications such as forecasting the generation of particular goods or services, technology mixes or the economic or environmental impacts that these may have. Geels (2002), with his multi-level perspective, rightly describes the diffusion of socio-technical systems as much more complex than simple substitution events represented by a set of coupled differential equations, involving niches, early uncoordinated innovations and transformations in the social context, seemingly precluding any modelling attempts. Despite this, it is remarkable that diffusion processes have been observed in a myriad of contexts to follow very closely logistic curves or the more general Lotka-Volterra system of equations (Farrell, 1993, Fisher and Pry, 1971, Lakka et al., 2013, Marchetti and Nakicenovic, 1978, Nakicenovic, 1986, Sharif and Kabir, 1976, Wilson, 2009, 2012, and many more), and that such simple patterns *emerge* out of the underlying complexity.

The problem can be simplified by restricting the analysis to the diffusion component, excluding the early erratic innovation process, assuming that new but established technologies permeate the landscape in dormant niches that could wake up and diffuse massively given the right environment, for instance with targeted policy. From then onwards the diffusion process, gaining momentum, becomes firmer and simpler to project quantitatively. Although the quantitative prediction of technology diffusion is inherently highly uncertain, in parts due to the actual evolutionary nature of technology evolution, it is nevertheless a highly worthwhile venture to undertake, particularly for the a climate change mitigation context, where it finds several important applications in energy intensive sectors (e.g. power generation, transport, industry).

While concepts of technology diffusion provide appropriate insight on the key dynamics involved, they have not been used significantly in the modelling literature beyond the empirical description of *past* data using the observed *pattern*, the logistic curve (Fisher and Pry, 1971, Marchetti and Nakicenovic, 1978, Nakicenovic, 1986, Sharif and Kabir, 1976, Wilson, 2009, 2012, and many more). With the exception of the author's model of the global power sector, *FTT:Power* (Mercure, 2012a), the process of technology diffusion has yet to be even considered in large scale cross-sectoral models such as those for energy systems modelling, where they could find significant practical applications, for instance in projecting energy use and greenhouse gas emissions. This is partly due to the fact that, while this type of theory suggests a system for forecasting technology or market evolution, such projections would rely on measured scaling parameters, which can be reliably measured only precisely in cases of older technologies where transitions have already occurred. Effectively, by the non-linear nature of the problem itself, obtaining such scaling parameters for new technologies for which forecasting would be critically important cannot be reliably done based on the small

amounts of available data.[1] This suggests that high gains could be generated if new insight could be found on how to obtain these parameters through other means than the empirical fitting of diffusion data, requiring to establish a quantitative theory to understand their nature. These parameters are *timescales*, and this suggests that their meaning is associated to the use, the advent and the demise of technology in time.

Ground work for building a theory of technology substitution and diffusion has been done in previous work. The first part consists with the definition of *FTT:Power*, a global model of technology diffusion, electricity generation and greenhouse gas emissions based on technology population dynamics and bounded rationality (Mercure, 2012a). The second part is an expansion of the theory of technology dynamics, separated from the associated problem of decision-making under bounded rationality for clarity, describing the structure of population dynamics equations in terms of growth and decommission rates (Mercure, 2012b). While these timescales are intuitively understood in terms of technology properties, they remain however exogenous to the model and their origin requires further explanation. This paper thus presents the theoretical elements necessary to construct a theory of technological change that uses scaling parameters that can be derived from the properties of the technologies and the industrial structures involved.

1.2. The Lotka-Volterra equation for empirical technology transitions

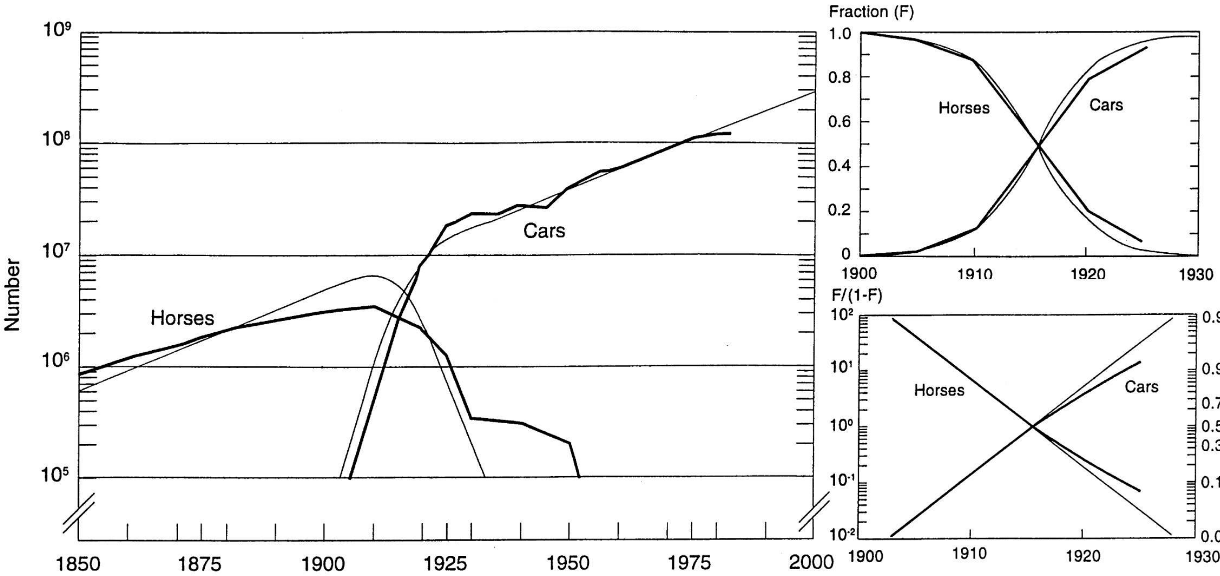

Figure 1: Transition from horses to cars in the 1920s (data originally from Nakicenovic 1986, graph taken from Grübler et al. 1999). (*Left*) Raw data on a semi-log axis. (*Top Right*) The data, when expressed as fractions of the total F, follows very closely logistic curves. (*Bottom Right*) This demonstrated by a transformation of the data of the form $F/(1-F)$ on a semi-log axis, which produces nearly linear trends.

The parallel between technology and ecology can be summarised as follows. Figure 1 presents the iconic data from the work of Nakicenovic (1986) for the transition between horses and cars that occurred in the 1920s. In this data, a transition is observed superimposed on an exponential growth of the number of vehicles. Through closer inspection, one observes that by dividing the numbers of horses and cars by the total number of transport units, functions reminiscent of logistic curves are observed which cross each other in around 1915. This is demonstrated to be a very accurate assessment by displaying the fractional data (using S here for market *Shares*) as $S/(1-S)$ on semilog axes, generating linear trends:

$$\log\left(\frac{S_1}{1-S_1}\right) = \alpha_{12}(t-t_0), \quad \log\left(\frac{S_2}{1-S_2}\right) = \alpha_{21}(t-t_0), \quad \alpha_{12} = -\alpha_{21}, \tag{1}$$

[1] E.g. fitting logistic curves requires data that spans at least beyond the inflexion point.

which corresponds to logistic forms,

$$S_1(t) = \frac{1}{1 + \exp\left(-\alpha_{12}(t - t_0)\right)}, \quad S_2(t) = 1 - S_1(t) = \frac{1}{1 + \exp\left(\alpha_{21}(t - t_0)\right)}. \tag{2}$$

Taking a differential form for these expressions, one obtains

$$\frac{dS_1}{dt} = \alpha_{12} S_1 (1 - S_1) = \alpha_{12} S_1 S_2, \quad \frac{dS_2}{dt} = \alpha_{21} S_2 S_1. \tag{3}$$

This example depicts the interaction occurring within a pair of technologies. Geels (2005) criticises the analysis of Nakicenovic (1986) by invoking the presence of two other important transport technologies, which have interacted with and influenced the development of petrol vehicles but have not pervaded the market, namely electric trams and bicycles. Effectively, in most cases of technology competition, it is nearly impossible to exclude the existence of a third interacting component, and a fourth and so on,[2]

$$\frac{dS_1}{dt} = \alpha_{12} S_1 S_2 + \alpha_{13} S_1 S_3 + \alpha_{14} S_1 S_4 + ... \quad \Rightarrow \quad \frac{dS_i}{dt} = \sum_j \alpha_{ij} S_i S_j, \tag{4}$$

generalising the theory to an arbitrary number of technologies interacting in the marketplace, with interaction time constants held in the antisymmetric matrix α_{ij}. In this form, it corresponds to the expression for the replicator dynamics well discussed in evolutionary game theory (Hofbauer and Sigmund, 1998) and evolutionary economics (Safarzynska and van den Bergh, 2010). It is also mathematically equivalent to the Lotka-Volterra system of differential equations for the numbers of individuals in a set of competing species when expressed in absolute numbers,

$$\frac{dN_i}{dt} = r_i \left[N_i - \sum_j \frac{\alpha_{ij} N_i N_j}{K} \right], \tag{5}$$

where the first term $r_i N_i$ is the *birth* of individuals with *birth rates* r_i, generating an exponential growth component, but the second term, negative, expresses both the interference of a specie with itself, when resources become scarce and individuals begin to compete, or the interference across species competing for the same resources. The new parameter K is the *carrying capacity* of the ecosystem, the number of individuals that the system can accommodate. In the technology context, the carrying capacity corresponds to the total number of units of technology supplying the demand for a service, or societal function, following a *demand led* economic assumption. Birth rates however have not yet been very clearly defined.

1.3. A demographic model of technological change

This paper presents a model of technological change that follows from the premise given above, using a replicator dynamics equation, with an additional attempt at providing meaning to its parameters (α_{ij}, r_i, K) in terms of information that can be obtained when dealing with technologies for which transitions have not yet occurred, making the parameters difficult to extract empirically from data. As I shall show, the replicator dynamics approach provides a clear definition of decision-making and bounded rationality, and divides the decision-making problem to the one of technology population dynamics, which can be treated separately. Thus the decision-making problem, while treated in the simplest possible form of bounded rationality ('profit-seeking as opposed to profit maximising', Nelson and Winter 1982) for clarity in my definition of FTT:Power (Mercure, 2012a), will be explored into more detail elsewhere. The crucial information required in modelling technology populations being the *timescales* of transitions, it must be strongly grounded in time-related concepts such as rates of capital investments and technology life expectancies, or in other words, technology birth and death processes. Since technologies *do possess* life expectancies and birth rates, it becomes unavoidable to consider making use of the massive but well trodden apparatus of age structured human demography. Several independent strands of demography exist, using either a continuous or a

[2]The perverse effect of using quantities relative to the total is that this method can easily lead to overlooking other competing technologies that only hold small market shares.

discrete form, all shown to be equivalent by Keyfitz (1977), of which I shall choose the continuous form in order to maintain a close relationship with the empirical work described above. Human demography in the continuous version corresponds in essence to an age structured form of single specie population dynamics. It provides an in-depth view of the process of population evolution through age specific stochastic birth and death events, using probabilities of giving birth and of dying for age tranches covering a whole lifetime, in other words, age specific birth and death rates. This provides demographers with much finer accuracy for population projections than the use of crude average birth and death rates. A system of competing species can also be described using an age structure, providing a similar accuracy improvement in the projection of interacting populations, if it is integrated to a competition model. I thus create here such a construction for technology dynamics, which, as I will show, demonstrate the origin of the shape of the technological change process due to its key property of *self-correlation* in time.

In contrast to demography, however, the birth of technology does not occur through pregnancy. Technology birth takes place inside of the industrial structure through the investment of financiers in production capital and labour, using for this the profits generated by the sales of these same technologies. Sales are the process by which population expansions can take place: if sales increase, the production capital and labour can be expanded, but if sales decrease, the production capital and labour must eventually shrink. The aim of this paper is thus to describe the process of birth, death and turnover of technology from first principles, based on assumed knowledge of the properties of technologies and their associated industries, but no knowledge of the structure of the anticipated technology transitions. This paper progresses as follows: I first describe independently the process of technology death, and then birth, and show how these generate a self correlation of population numbers in time. I then construct an age structured model of interacting technology populations. In a particular approximation under specific constraints, this model is shown to reduce to a Lotka-Volterra competition system. This process uncovers the origin of the parameters of the Lotka-Volterra equation, grounded in demographic theory. While death rates are nearly obvious, the structure of technology birth rates is more subtle, and this calculation generates insight on their meaning in terms of industrial dynamics. This is explored at this point. I then close the analysis with a discussion in terms of the multi-level perspective, defining a *demographic phase* of technological change, which takes place after innovation generates coherent seeds of technology diffusion that reside in technology niches. I then present a description of the practical use of this theory in applications such as energy systems modelling, but also of the general understanding of technology transitions.

2. The birth and death of technology

2.1. *The destruction process*

The death of technology units can occur in different ways with different probabilities. For example, in the transport sector, vehicles can be taken out of the system through fatal accidents, or by random failures during their lifetime, or perhaps by economic decisions due to the cost of maintenance increasing towards old age.[3] These processes have different probabilities of occurring as functions of vehicle age. For a technology of type or brand i, taking the probability of destruction at age a as $p_i(a)\Delta a$, and the number $n_i(a,t')\Delta t'$ of technology units produced between year t' and $t' + \Delta t'$ (or age interval Δa),[4] the change in this age distribution $\Delta n(a,t)$ of technology units during an ageing interval Δa due to destructions is

$$\Delta n_i(a,t')\Delta t' = -p_i(a) n_i(a,t')\Delta t' \Delta a \tag{6}$$

In the continuous limit ($\Delta a \to 0$), this solves to

$$n_i(a,t')\Delta t' = n_i(0,t')\ell_i(a)\Delta t', \quad \ell_i(a) = \exp\left(-\int_0^a p_i(a')da'\right). \tag{7}$$

$\ell_i(a)$ is the common demographic *survival function*, while $p_i(a)$ is the instantaneous *force of death* (see for instance Keyfitz, 1977). This is depicted in fig. 2. These are normally derived in demography from life tables where individuals are traced during their lifetime from birth until death. The various processes of technology death can be associated to components in $\ell_i(a)$. Accidents normally have a constant force of death, and therefore give $\ell_i(a)$ a simple exponential

[3] The existence of *sunk costs* imply the existence of a non-zero life expectancy.

[4] E.g. the number of 2003 Citroen C3 currently 10 years old.

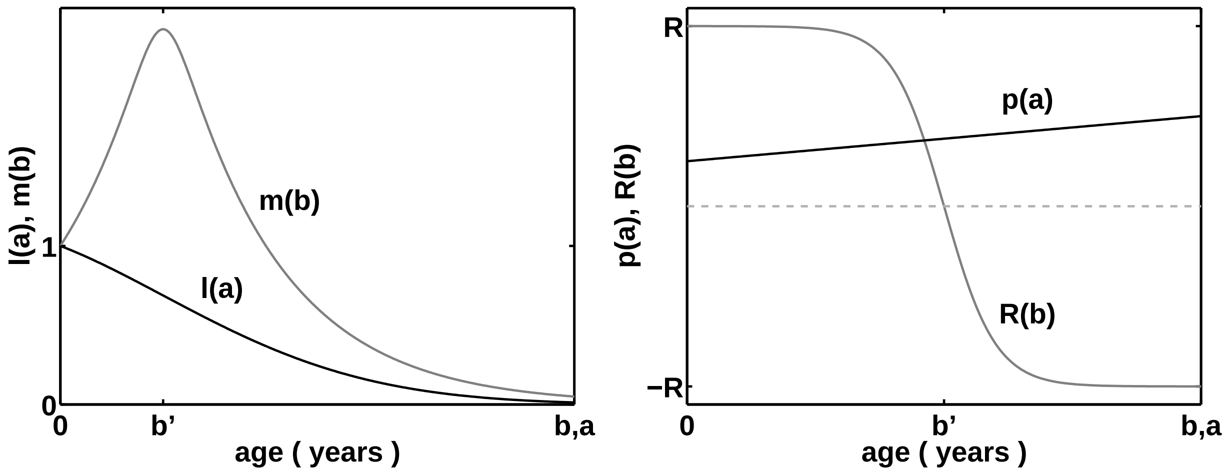

Figure 2: Depiction of the survival function $\ell(a)$ and the birth function $m(b)$ (*left*), which respectively stem from the instantaneous force of death $p(a)$ and the reinvestment schedule $R(b)$ (*right*). The reinvestment schedule becomes negative when the maintenance of old production capacity exceeds its income and it is gradually taken out of operation.

form. Meanwhile, scrapping due to failures tend to occur later during technology life, with increasing values of $p(a)$. Thus $\ell(a)$ can be written as

$$p(a) = \frac{1}{\tau_1} + \frac{a}{\tau_2^2} + \frac{a^2}{\tau_3^3} + ..., \quad \ell(a) = \exp\left(-\frac{a}{\tau_1} - \frac{a^2}{2\tau_2^2} - \frac{a^3}{3\tau_3^3} - ...\right), \tag{8}$$

each term corresponding to different processes with respective timescales τ_n. If accidents dominate the destruction process, then $\ell_i(a)$ should take predominantly an exponential form, while if the probability of failures dominates and increases approximately linearly with age, $\ell_i(a)$ takes the form of a gaussian, and so on. The survival of transport vehicles in the USA was shown to follow approximately a mixture of τ_1 and τ_2 processes (ORNL, 2012, and references therein). While $\ell_i(a)$ expresses the probability of a technology unit to remain in use until age a, the negative of its derivative expresses the probability of destruction at age a. The life expectancy μ_i is defined as

$$\mu_i = -\int_0^\infty a\frac{d\ell_i(a)}{da}da = \int_0^\infty \ell_i(a)da, \tag{9}$$

where the last expression above is obtained from the previous by integration by parts. In the simple case of death dominated by accidents, $\mu = \tau_1$ and units of a particular age tranche decrease in numbers exponentially at a rate equal to the life expectancy.

Every year $t = t' + a$, a certain number of deaths $d_i(t)$ occur, technology units that are scrapped in some way or another, while a number $\xi_i(t)$ of units are sold, both changing the total number of units in use,

$$\frac{\Delta N_i}{\Delta t} = \xi_i(t) - d_i(t). \tag{10}$$

While sales generate units of age zero, deaths decrease the number of units of all ages,

$$\frac{dn_i(a,t')}{dt}\Delta t' = n_i(0,t')\frac{d\ell_i(a)}{da}\Delta t', \quad n_i(0,t') = \xi_i(t') \tag{11}$$

where for each age tranche between a and Δa (or production year between t' and $t' + \Delta t'$), the number of deaths depend on the probability of destruction as well as on the number of units of that age (or production year) remaining, which decreases every year. The number of units in each age tranche originates from sales which happened in year t' (a years ago). Thus, in the continuous limit $\Delta t' \to 0$, while the total number of units at time t depends on the number of units sold in the past that still remain at time t,

$$N_i(t) = \int_{-\infty}^t \xi_i(t')\ell_i(a)dt' = \int_0^\infty \xi_i(t-a)\ell_i(a)da, \tag{12}$$

the reduction in the number of units at year t due to deaths is the sum of the number of units that remain in each age tranche and their probability of being destroyed precisely in year t,

$$d_i(t) = -\int_0^\infty \xi_i(t-a)\frac{d\ell_i(a)}{da}da. \tag{13}$$

Both these expressions are *convolutions* of past sales with either the survival function or the death probability. If sales $\xi_i(t)$ are related in any way to the existing number of units, this produces a *self-correlation* of the number of units with itself in past years. I will show later that this is effectively the case, which restricts how fast the total number of units can change in the system.[5]

2.2. *The birth of technology*

The number of units of technology that can be built in a time span depends on the production capital and labour available at that time. The fraction of production capacity[6] for technology i, $\delta N_i(b,t')\Delta t'$, built in years between t' and $t' + \Delta t'$ stems from the construction of new production lines or the creation of firms in that year, as well as those created in all previous years b. After its construction, production capital will generate funds from the sale of the units it produces, which are reinvested into expanding the total production capacity by building new production lines. This production capital however also has a lifespan, a probability of failure and thus a survival function. Therefore, the production capacity increase $\Delta\delta N_i(b,t')\Delta t'$ generated by a production line of capacity $\delta N_i(b,t')\Delta t'$ of age b producing technology units of type i is

$$\Delta\delta N_i(b,t')\Delta t' = R_i(b)\delta N_i(b,t')\Delta t'\Delta b, \tag{14}$$

where $R_i(b)$ is meant to represent the reinvestment schedule. $R_i(b)$ is the rate of increase of the production capacity due to sales $\xi_i(t')$ in year t', b years ago, and will be described into more detail further. The size of an age tranche of the industry as it evolves with age b is

$$\delta N_i(b,t')\Delta t' = \delta N_i(0,t')m_i(b)\Delta t', \quad m_i(b) = \exp\left(\int_0^\infty R_i(b)db\right). \tag{15}$$

As opposed to $\ell_i(a)$, $m_i(b)$ it is not a decreasing function, but it increases initially, as the production capacity generated expands, before decreasing in later years when old production lines get decommissioned, its sales generating less funds for creating new production capacity than it costs to maintain itself (depicted in fig. 2). It must be an integrable function, the area under which $\Phi_i = \int_0^\infty m_i(b)db$ converges. Furthermore, it is not a normalised function as is $-d\ell_i(a)/da$, because this process generates an increase in the total production capacity and, as we show below, $R(0)\Phi_i > 1$. [7]

With reinvestment schedule in the first year $R(0)$, the creation of new production capacity in year t' using the funds generated by sales at that time $t' = t - b$ is

$$\delta N_i(0,t')\Delta t' = R_i(0)\xi_i(t')\Delta t'. \tag{16}$$

Therefore, in the continuous limit, the total production capacity at year t is the sum of all production capacity created in previous years from sales:

$$\delta N_i(t) = R_i(0)\int_0^\infty \xi_i(t-b)m_i(b)db \tag{17}$$

This is another convolution, the second half of the problem, which generates, if the number of units is related to sales, to a second autocorrelation in the number of units. In the particular case where the production capacity is fully used and sales are exactly equal to the production capacity, the production capacity increases exponentially, at a rate that has yet to be evaluated. This expression is very similar in form to the 'renewal equation' in demography (see for

[5] Note that in contrast to the birth function that I shall define further, $\ell_i(a)$ must be a strictly decreasing function of age otherwise dead units would come back to life.

[6] The production capacity is taken here to include both production capital and labour for simplicity.

[7] Otherwise sales generate less production capacity than was used to produce the units sold.

instance Kot, 2001, Metz and Diekmann, 1980), expressing a time correlation of past birth rates with later birth rates, with a maternity function $m(a)$ and a survival function $\ell(a)$,

$$B(t) = \int_0^\infty B(t-a)m(a)\ell(a)da. \tag{18}$$

In contrast to biology, for technologies, since units do not undergo pregnancy at age a with probability $m(a)$ and probability of survival to that age $\ell(a)$, the equation does not feature a product of the survival function and the maternity function, and the meaning of the birth function is different.[8]

3. An age structured model of technological change

3.1. Destructions replaced by constructions

In a model of competition, several technologies compete in the marketplace to produce the same good or service. In many contexts, the consumption of that service is completely indifferent as to which technology has produced it, for example with transport services provided by vehicles with petrol, electric or diesel engines. In that case, one can assume that the total demand is independent of the composition of the technology mix that supplies this demand. In this context, building a model of competition requires determining the flow of units from one category of technology to another, and then generalising this reasoning to create a closed system that conserves the number of units. This was done in earlier work (Mercure, 2012b), which demonstrated how one can construct and understand the structure of a model of technology substitution, summarised here.

In this model, the choice of consumers or investors was separated from the diffusion process, for clarity. Choices of consumers or investors is taken to mean what choices would be made if all options were available but not necessarily well known, and these are defined in terms of preferences in the comparison of each possible pairs of technologies. Given that despite the first choice of consumers or investors, those may not necessarily be available in every individual situations, consumers or investors may have to content themselves with their second or third choice.

Assuming that units of any age are replaced by new ones when they are removed from the system through destruction,[9] following this approach, I evaluate the number of units flowing from one arbitrary technology category j towards another category i. For this, I start with the total number of destructions in all vehicle categories, and find how many of those belong to category j. Out of those destructions in j, I evaluate those that were chosen by consumers to be replaced by technology i, a choice process described below.

Of these, only a fraction can be produced. This statement is not obvious, and I shall attempt to explain it here. The number of individual situations where a choice is made is large. The production capacity of a particular technology may not necessarily be able to supply the demand in every one of these situations, were the consumers to all simultaneously choose this technology, if the production capacity is not high enough to generate this number. Therefore, in a certain number of these situations, the option will simply not be available, and consumers will have to choose between the remaining options despite their best preference. The fraction of instances where this choice will be available with respect to the total number of choices being made corresponds to the fraction of production capacity of this technology with respect to the total production capacity. This can be understood through an analogy involving an ensemble of shops with a number of competing products on their shelves. Given the production capacity of each product's respective industry, most of the shops may not be able to stock units of all competing products, restricting the local choice of the customers who go to those shops. When customers have equal preferences for all products, the relative probability of the average customer choosing particular products corresponds to the *average* composition of the product choice in the ensemble of shops, which itself corresponds to relative production capacity of each product with respect to the total. Thus the fraction of units of technology j, chosen to be replaced by technology i, that can actually be replaced by units of i corresponds to the fraction of the total production capacity that produces technology i.

[8]Although some species, such as ants and bees, have individuals that do not undergo pregnancy but do contribute to the survival of the colony through other means, and there the dynamics may be somewhat closer to those described in this paper.

[9]i.e. this only means that old units do not come back to life. Note that the nature of the ownership of these technology units, and whether they change ownership, is not important, which enables to make complete abstraction of second-hand markets.

As presented in earlier work (Mercure, 2012a,b), I define consumer preferences in terms of the pairwise comparison of technologies as follows: when investors wishing to replace old units are faced with a choice between two technologies, I take a probabilistic approach where a fraction F_{ij} of consumers choose one technology while the rest F_{ji} choose the other, and therefore $F_{ij} + F_{ji} = 1$. This choice need not be fixed in time, since of course preferences and costs continuously change, in particular with technological learning. A list of technology is then explored in terms of this matrix by performing all possible pairwise comparisons. Thus, if some investors on average prefer j to i, some of them may still prefer a third option k to j, and so on. This generates an exhaustive (probabilistic) ranking of technologies based upon which flows in all directions can be evaluated. Thus the overall result of choices generates the ultimate driver for diffusion

The flow of units from categories j to i is summarised by the following, which is read from right to left:

$$\Delta N_{j\to i} = \begin{pmatrix} Fraction\,of\,constructions \\ belonging\,to\,i \end{pmatrix}_i \times \begin{pmatrix} Consumer \\ preferences \end{pmatrix}_{ij} \times \begin{pmatrix} Fraction\,of\,destructions \\ belonging\,to\,j \end{pmatrix}_j \times \begin{pmatrix} Total \\ Destructions \end{pmatrix}_{tot}. \quad (19)$$

Accounting for the total change originating from flows in both directions, ΔN_{ij}, the sum of which generates the total change produced ΔN_i by the combined interaction between one technology and every other is

$$\Delta N_{ij} = \Delta N_{j\to i} - \Delta N_{i\to j}, \quad \Delta N_i = \sum_j \Delta N_{j\to i} - \Delta N_{i\to j}. \quad (20)$$

This construction enables to define an age structured demographic model of technology.

3.2. The age structured model

Eq. 19 can be written in terms of the production capacity $\delta N_i(t)$ and deaths $d_i(t)$:

$$\Delta N_{j\to i} = \left(\frac{\delta N_i(t)}{\sum_k \delta N_k(t)}\right) \times F_{ij} \times \left(\frac{d_i(t)}{\sum_k d_k(t)}\right) \times \left(\sum_k d_k(t)\right)\Delta t. \quad (21)$$

The production capacities and death numbers at time t can be replaced by convolutions of the sales:

$$\Delta N_{j\to i} = \left(\frac{R_i \int_0^\infty \xi_i(t-b)m_i(b)db}{\sum_k R_k \int_0^\infty \xi_k(t-b)m_k(b)db}\right) \times F_{ij} \times \left(\frac{\int_0^\infty \xi_j(t-a)\frac{d\ell_j(a)}{da}da}{\sum_k \int_0^\infty \xi_k(t-a)\frac{d\ell_k(a)}{da}da}\right) \times \left(\sum_k \int_0^\infty \xi_k(t-a)\frac{d\ell_k(a)}{da}da\right)\Delta t. \quad (22)$$

Note the symmetry between the production side and the destruction side of this equation. There is, effectively, a high similarity between both processes. The difference however is fundamental: while $\ell(a)$ is a strictly decreasing function of age, $m(b)$ both increases and decreases. The decreasing nature of $\ell(a)$ generates destruction, while the increasing part of $m(b)$ generates production. However, in order not to have an indefinitely increasing production capacity, $m(b)$ must also decrease again at high values of b, maintaining the function integrable, generating decreases in the production capacity when sales decrease, reflecting the gradual depreciation and wearing out of the production capital.

Eq. 20 with eq. 22 provides an expression for exchanges of units between categories in absolute numbers. However, the total number, the carrying capacity, could be changing, requiring either units that are brought in that do not replace deaths, or deaths that are not replaced. In the more common case of a total number K increasing, this is met by technology production following the relative production capacity:

$$\Delta N_i^{\uparrow} = \left(\frac{R_i \int_0^\infty \xi_i(t-b)m_i(b)db}{\sum_k R_k \int_0^\infty \xi_k(t-b)m_k(b)db}\right) \times \left(\frac{\Delta K}{\Delta t}\right)\Delta t, \quad (23)$$

where choices need not be involved.[10] Meanwhile in the second less common case, the decrease in K is met by the relative rate of deaths,

$$\Delta N_i^{\downarrow} = \left(\frac{\int_0^\infty \xi_i(t-a)\frac{d\ell_i(a)}{da}da}{\sum_k \int_0^\infty \xi_k(t-a)\frac{d\ell_k(a)}{da}da}\right) \times \left(\frac{\Delta K}{\Delta t}\right)\Delta t. \quad (24)$$

[10] Adding here a factor F_{ij} can be done but is secondary: even if new units are not chosen exchanges can occur through the exchange term.

Assembling these expressions together, one obtains an expression too large to write here, summarised by

$$\Delta N_i = \sum_j \Delta N_{ij} + \Delta N_i^{\uparrow} \quad \text{or} \quad \Delta N_i = \sum_j \Delta N_{ij} + \Delta N_i^{\downarrow}. \tag{25}$$

When terms are replaced in eq 25, the resulting large expression corresponds to the demographic model of technology expressed in terms of the full sales history. This model can also be expressed uniquely in terms of sales, where $\xi_i(t) = \sum_j \Delta N_{j\to i} + \Delta N_i^{\uparrow}$:

$$\xi_i(t) = \sum_j \left(\frac{R_i \int_0^\infty \xi_i(t-b) m_i(b) db}{\sum_k R_k \int_0^\infty \xi_k(t-b) m_k(b) db} \right) \times F_{ij} \times \left(\int_0^\infty \xi_j(t-a) \frac{d\ell_j(a)}{da} da \right) + \left(\frac{R_i \int_0^\infty \xi_i(t-b) m_i(b) db}{\sum_k R_k \int_0^\infty \xi_k(t-b) m_k(b) db} \right) \times \left(\frac{\Delta K}{\Delta t} \right) \tag{26}$$

This expresses how sales at any time are constrained by *sales in the past* through convolutions, generating self-correlations of the sales, in other words present sales correlated to the amounts of sales in the past, within and between categories. Since sales are autocorrelated, and that the addition of units corresponds to sales and removals to deaths, it implies that the absolute numbers of units are self-correlated as well. Therefore, changes in the numbers of units cannot change faster than is allowed by the self-correlation, which as we demonstrate next, is given by the width in time of the functions $\ell_i(a)$ and $m_i(b)$. Going any further requires evaluating all the convolutions, which requires knowledge on sales $\xi_i(t)$, survival functions $\ell_i(a)$ and birth functions $m_i(b)$.

3.3. *Convolutions*

Eq. 25 in its full form, or alternatively eq. 26, appear rather unconstrained and uninstructive. Since they are recurrent, these equations are more constrained in the behaviour of their possible solutions than they seem. Eq. 25 in full form expresses technological change between technology categories in terms of respective sales of those technologies. These sales are convolved with the functions $m(b)$ and $d\ell(a)/da$. It is well known in signal processing theory that convolutions of functions with bounded kernels (fig. 2 *left*) yield slightly modified functions that are *smoothed* with respect to the original, where (high frequency) sharp changes have been suppressed.[11] The 'cutoff' value at which frequencies are suppressed, the sharpness limit, corresponds to the *width* in time of the kernel.[12] This is also the *correlation length* of the smoothed function. For symmetrical normalised[13] kernels of similar widths, the convolution of a function leads to very similar results since a similar frequency cutoff occurs, even if the kernels have different shapes, and the same high frequencies are similarly suppressed. If a kernel is not normalised, it either amplifies the signal (its integral is greater than one) or damps it (its integral lower than one). If both kernels are not normalised but of similar width, the convolutions will yield results which are close to multiples of each other, with proportionality factor the relative area under the kernels. Finally, if the kernels are not symmetrical functions, as is the case here, a time offset may appear between the two results.

The first kernel, the birth function $m_j(b)$, has the following property,

$$R_j(0) \int_0^\infty m_j(b) db = R_j(0)\Phi_j > 1, \tag{27}$$

which reflects the growth of the production capacity through reinvestment. The second kernel, $\ell_i(a)$, is normalised by definition (expressing a eventual but certain death):

$$\int_0^\infty -\frac{d\ell_j(a)}{da} da = 1. \tag{28}$$

In the case of a uniform probability of death through time (a constant instantaneous force of death $p_i = 1/\tau_i$), $\ell(a)$ would have the form of an exponential decay of width $\mu_i = \tau_i$, the statistical life expectancy of units, and

$$-\frac{d\ell(a)}{da} = \frac{\ell(a)}{\tau_i}. \tag{29}$$

[11] i.e. a 'low-pass' filter.

[12] In this case both $m(b)$ and $d\ell(a)/da$; the wider the kernel, the lower the frequency cutoff and the more smoothing occurs.

[13] The area under a normalised kernel equals one.

For survival functions of slightly different shapes, this may still hold approximately:

$$\ell_i(a) = \exp\left(\sum_k a^k/\tau_{ik}^k\right), \quad \frac{d\ell_i(a)}{da} = \ell_i(a)\left(\frac{1}{\tau_i} + \frac{a}{\tau_{i2}^2} + \frac{a^2}{\tau_{i3}^3} + ...\right), \tag{30}$$

with terms of ever decreasing importance. Taking this approximation, the number of units of j coming to death can be approximated to a simple quantity

$$\int_0^\infty \xi_j(t-a)\frac{d\ell_j(a)}{da}da \simeq \frac{1}{\tau_j}\int_0^\infty \xi_j(t-a)\ell_j(a)da = \frac{N_j(t)}{\tau_j}, \tag{31}$$

where as was shown with eq. 13, the second integral corresponds to the actual number $N_j(t)$ of units still in use.

In a case where the width of the second kernel, the birth function $m_i(b)$, is similar to the width of the survival function $\ell_i(a)$ (or alternatively the death function $-\frac{d\ell_i(a)}{da}$), the convolution of sales by one or the other of these functions will not be very different, but rather approximately proportional. Conversely, if the widths are very different, they *cannot* in any way be proportional or even similar. The width of the birth function is related to the survival function of the capital and labour used for production, the production lines, which may have, in some situations, a similar time scale. Assuming that this is so, and since $-\frac{d\ell_i(a)}{da}$ is normalised, then the convolutions with $m_i(b)$ and $-\frac{d\ell_i(a)}{da}$ are approximately proportional, and the proportionality factor is $R_i\Phi_i$ (with $R_i = R_i(0)$):

$$R_i\int_0^\infty \xi_i(t-b)m_i(b)db \simeq -R_i\Phi_i\int_0^\infty \xi_i(t-a-t_0)\frac{d\ell_i(a)}{da}da \simeq R_i\Phi_i\frac{N_i(t-t_0)}{\tau_i} \simeq \frac{N_i(t-t_0)}{t_i}, \tag{32}$$

where t_i is a new timescale relating to the growth rate of the production capacity. The factor $R_i\Phi_i > 1$ originates from the fact that production lines generate more units than was required to generate the funds used for their own construction. Therefore one has that $t_i = \tau_i/R_i\Phi_i$, a shorter timescale for the growth of production than for destruction. This *must* be the case otherwise the industry does not regenerate some of its components which gradually go out of business by producing more slowly than units get destroyed. The additional time offset t_0 appears due to the differing degree of asymmetry of the kernels, moving the functions $N_i(t)$ forwards or backwards in time slightly with respect to one another. The degree of symmetry difference between the kernels is limited and the time offset cannot be very large; in fact it is much smaller than the width of the kernels, and I omit it henceforth.

3.4. *Approximating the Lotka-Volterra equation*

These approximations can be directly used to significantly simplify eqns. 22 and 25. Replacing each convolution by its associated approximation,

$$\Delta N_{j\to i} = \left(\frac{\frac{N_i(t)}{t_i}}{\sum_k \frac{N_k(t)}{t_k}}\right)\times F_{ij}\times\left(\frac{\frac{N_j(t)}{\tau_j}}{\sum_l \frac{N_l(t)}{\tau_l}}\right)\times\left(\sum_m \frac{N_m(t)}{\tau_m}\right)\Delta t, \tag{33}$$

which is exactly the result obtained previously (Mercure, 2012b). Defining the average frequencies $\bar{t}^{-1}$ and $\bar{\tau}^{-1}$,

$$\frac{1}{\bar{t}} = \frac{1}{K}\sum_k \frac{N_k(t)}{t_k} \quad \text{and} \quad \frac{1}{\bar{\tau}} = \frac{1}{K}\sum_l \frac{N_l(t)}{\tau_l}, \tag{34}$$

the flow becomes

$$\Delta N_{j\to i} = \left(\frac{\bar{t}}{t_i}\frac{N_i(t)}{K}\right)\times F_{ij}\times\left(\frac{\bar{\tau}}{\tau_j}\frac{N_j(t)}{K}\right)\times\left(\frac{K}{\bar{\tau}}\right)\Delta t, \tag{35}$$

while the term concerning increases in K becomes

$$\Delta N_i^{\uparrow} = \left(\frac{\bar{t}}{t_j}\frac{N_i(t)}{K}\right)\times\left(\frac{\Delta K}{\Delta t}\right)\Delta t \tag{36}$$

Using a new matrix A_{ij} summarising all time constants, the total changes become

$$\Delta N_i = \left[\sum_j \frac{N_i N_j}{K} \left(A_{ij} F_{ij} - A_{ji} F_{ji} \right) + \frac{\bar{t}}{t_i} \frac{N_i(t)}{K} \left(\frac{\Delta K}{\Delta t} \right) \right] \frac{\Delta t}{\bar{\tau}}, \quad A_{ij} = \frac{\bar{t}\bar{\tau}}{t_i \tau_j} \tag{37}$$

This is the Lotka-Volterra equation again. The replicator dynamics equation 4 can be obtained using the chain derivative:

$$\frac{dN_i}{dt} = K \frac{dS_i}{dt} + S_i \frac{dK}{dt}, \tag{38}$$

and obtain

$$\Delta S_i = \left[\sum_j \frac{1}{\bar{\tau}} S_i S_j \left(A_{ij} S_{ij} - A_{ji} S_{ji} \right) + \frac{\bar{t}}{t_i} S_i \frac{\Delta K}{dt} - S_i \frac{\Delta K}{dt} \right] \Delta t, \tag{39}$$

which, if the t_i do not differ significantly from the average $\bar{t}$, reduces approximately to

$$\Delta S_i = \sum_j \frac{1}{\bar{\tau}} S_i S_j \left(A_{ij} S_{ij} - A_{ji} S_{ji} \right) \Delta t. \tag{40}$$

This replicator dynamics equation has an antisymmetric exchange matrix $\alpha_{ij} = A_{ij} S_{ij} - A_{ji} S_{ji}$. This equation is the one used in Mercure (2012a) to evaluate changes in the power sector technology mix of *FTT:Power*. Note that the two matrices used, A_{ij} and F_{ij}, separate the demography from the decision-making processes into independent considerations. It thus enables to introduce various types of decision-making assumptions into the same model of technology dynamics (e.g. investor behaviour under uncertainty or adaptive dynamics).

4. Interpretation of the Lotka-Volterra scaling parameters

4.1. Constraints and applicability of the Lotka-Volterra model

This calculation determines the constraints under which the general demographic model of technological change falls back onto the more specific empirical Lotka-Volterra equation:

1. The birth and death functions must have similar approximate widths in time
2. The dominant destruction mechanism must be close to a simple exponential
3. The area under the birth function for technology i, Φ_i, times the reinvestment schedule R_i, must be greater than one for the technology type to be able to replicate itself.

One then finds that $R_i \Phi_i$ determines the growth time constant in terms of the lifetime: $t_i = \tau_i / R_i \Phi_i$, where

$$\frac{1}{t_i} = \frac{R_i(0)}{\tau_i} \int_0^\infty \exp\left(\int_0^b R_i(b') db' \right) db, \tag{41}$$

with $R_i(b)$ a rate of reinvestment schedule in a production line of age b. Thus it is unsurprisingly the profile of reinvestment that determines the magnitude of the rate of growth of the production capacity t_i. Thus in order for the industry to grow, such that $t_i < \tau_i$, one must have that

$$R_i \Phi_i = R_i \int_0^\infty \exp\left(\int_0^b R_i(b') db' \right) db > 1. \tag{42}$$

But what do $R(b)$ and $m(b)$ mean? $R(b)$ is related to the reinvestment of a fraction of the income of sales back into a production line as it ages, or alternatively in a finance context, the ability of borrowing money given the current success of the firm defined by the magnitude of its sales. There comes a point where particular production capital is not kept working but is gradually taken down to be replaced by something else. However, during its lifetime, this capital will have produced a large number of units each of which the sale will have generated income, a fraction of which will have been reinvested and will have expanded its capacity, such that it will have been able to produce more units, the

greater amount sales of which will have generated ever more income and so on. In a situation of unconstrained sales, this leads to a simple exponential growth. However, the production capacity does not work forever but depreciates and ages, preventing an exponential increase to infinity of its capacity unless sales do the same, decreasing if sales decrease. While in biology the birth function $m(b)$ is the age specific average number of offspring per female in her limited lifetime, in the technology context it is a reflection of the multiplier effect of sales onto the production capacity and of its limited lifetime, or alternatively, the multiplier effect of the cumulative success of a firm onto its ability to borrow money for expanding its capacity.

The conditions listed in this section generate strict constraints that provide insight determining which systems may be modelled using the Lotka-Volterra set of equations (LVEs):

1. If the kernels have very different widths, the LVEs are not appropriate. This occurs if the life expectancy of the production capital is much longer or much shorter than the life expectancy of the produced units. [14]
2. The LVEs assume death rates proportional to the numbers of existing units. This approximation is equivalent to a uniform probability of destruction at any time. For technologies where the destruction factor is strongly limited by something else than accidents or random technical failures, a more elaborate model should be used.[15]
3. The producing firms must have an intended propensity towards expansion, and must reinvest enough profits to expand their production capacity, which will only decline if sales decline due to a lack of competitiveness. In a case where a firm has made a decision not to maintain a technology under production despite that it is profitable, the Lotka-Volterra model breaks down.

This clarifies under which constraints the Lotka-Volterra model can be used. In evaluating the evolution of the market shares of firms for a particular market, the technology unit used in the Lotka-Volterra equation is crucial. The units must be service producing technologies (e.g. ovens, power stations, vehicles, lighting devices), not the service itself (e.g. a piece of bread, a kWh, a transport service, light) or long-lived infrastructure (e.g. houses or buildings, roads, airports, sets of transmission lines, bridges) likely to be maintained for lengths of time beyond foreseeable future.

4.2. Interpretation of scaling parameters

The interpretation of τ_i is the life expectancy of technology units, which may be calculated using a survival function, and must be considered as a statistical lifetime. As detailed in Mercure (2012b), it gives rise to the technology turnover $\overline{\tau}$, a constant controlling the overall rate of change.[16]

The interpretation of $t_i = \tau_i / R_i \Phi_i$ and $\overline{t}$ is more subtle. $R_i(b)$ is the reinvestment schedule, expressing a rate of production capacity increase that can be obtained from the fraction of profits on sales that is reinvested, the capital cost of new production capacity and the operation and maintenance cost of existing production capacity.

$$R_i(b) = \frac{Income_i(b) \times Fraction_i \quad [\$/unit\, Tech./year]}{CapitalCost_i(b) + O\&M_i(b) \quad [\$/unit\, Prod.\, Cap.]} \tag{43}$$

$R_i(b)$ is positive in early years and becomes negative in later years, where the operation and maintenance costs of ageing production capacity exceeds the income on sales and profits become negative. $R_i(b)$ need not be greater than one, as it is an accumulation of capital faster than its depreciation that generates an increase in production capacity, which is expressed by $m(b)$, or alternatively, Φ_i. Meanwhile, the parameter Φ_i expresses the multiplier effect of industry growth starting from individual units of production capacity. Thus, the longer the life of production capacity, or the higher the reinvestment fraction, the faster the growth pace of the industry becomes, expressed by a shorter growth timescale t_i. $\overline{t}^{-1}$ expresses the average rate of growth across the sector. The rate of profit making scales with the number of units produced per unit time. For long construction projects, t_i^{-1} can be seen as proportional to the construction time.

[14] E.g. the mobile phone industry, in which phones have very short lifetimes, or infrastructure industries where the capital, e.g. houses, roads and bridges, have much longer lifetimes than the firms building them, potentially maintained forever.

[15] E.g. mobile phones, where the decision to scrap is more related to the contract structure than to the wearing out of the product

[16] Note that $\overline{\tau}$ need not be *constant* in time, but evolves if the relative frequency of technologies with different life expectancies change. In a case where shorter lived technologies penetrate the market, $\overline{\tau}$ shortens and the system accelerates, indicating a faster average turnover.

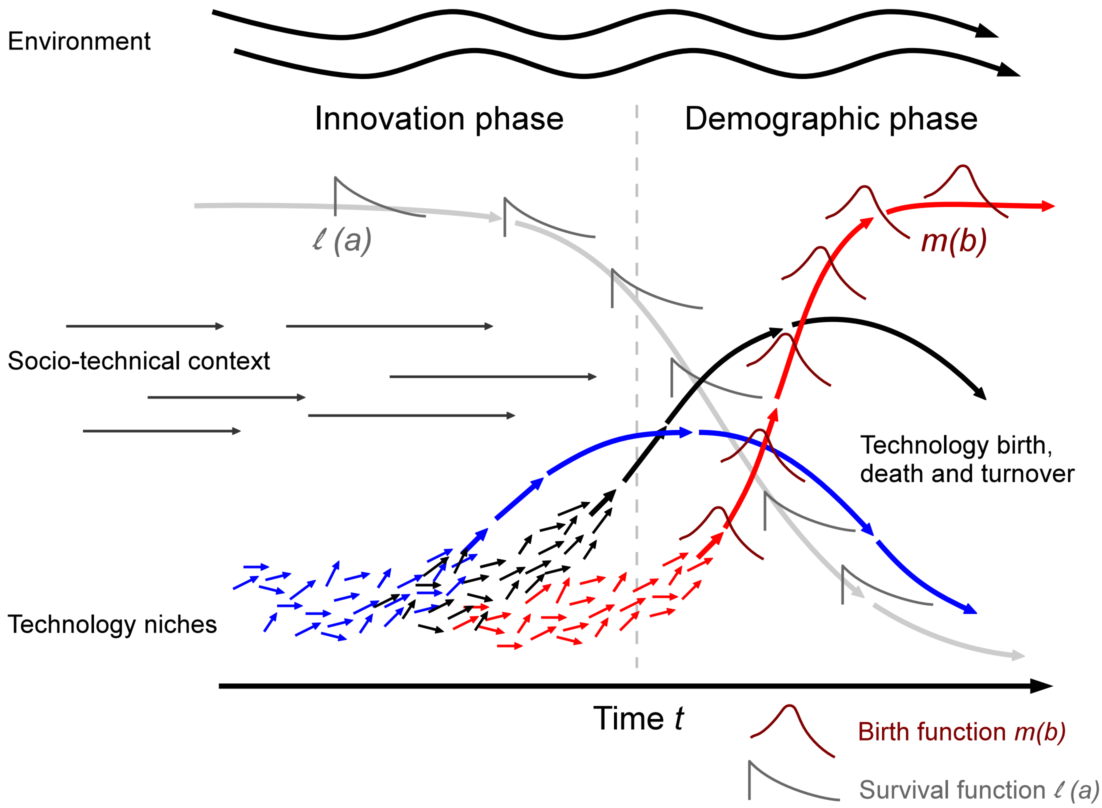

Figure 3: Illustration of the demographic phase of technology transitions, adapted from Geels (2002). Small arrows represent small incremental erratic innovations, during the innovation phase, which gradually gain coordination and momentum before diffusion takes place, entering the demographic phase. One technology is in decline (gray), disappearing at a maximum rate related to its survival function $\ell(a)$. One technology initially gains market shares at the expense of the one declining (blue), but is in time beaten in the race by another (black), which in turn is overtaken by yet another technology (red). The maximum growth rate is related to the birth function $m(b)$. The socio-technical context, consumer preferences and the environment generate selection mechanisms driving market share exchanges between technologies.

As indicated in detail in Mercure (2012b), it is interesting to note that growth timescales in eq. 40 only ever appear as ratios with the average $\bar{t}$. Therefore, if within a sector, it highlights that differences between the rates of growth of the production capacity of various technologies are important, their absolute value cancels out and does not influence the overall rate of change, which is controlled by the average scrapping rate $\bar{\tau}$. In particular, the rate of return on investment scales with the rate of construction (the inverse of the lead time) where a firm may in some cases be operating on specific construction projects without immediate income. As for example may happen in the power sector, this may restrict the rate of growth (the parameter t_i^{-1}) proportionally to the rate of construction, and that faster growing types of technologies can fill in gaps in the market faster than slow growing firms. Where other parameters are similar for different technologies, the ratio $\bar{t}/t_i$ becomes proportional to the ratio of the inverse lead time (rate of construction) with respect to the average inverse lead time (average rate of construction). This can simplify dramatically the parameterisation of a Lotka-Volterra model down to lifetimes and lead times while remaining a credible model.

4.3. *Example with a constant reinvestment schedule*

These properties can be clarified by using the simplest possible example, a situation perhaps of seemingly unrealistic nature, but which depicts the nature of the results given above. Assuming that the reinvestment schedule is of a

constant value R_i up to an age $\hat{b}$, after which it becomes $-R_i$:

$$R_i(b) = \begin{cases} R_i, & 0 \leq b \leq \hat{b}_i \\ -R_i, & b > \hat{b}_i \end{cases}, \qquad m_i(b) = \begin{cases} e^{R_i b}, & 0 \leq b \leq \hat{b}_i \\ e^{R_i(2\hat{b}_i - b)}, & b > \hat{b}_i \end{cases},$$

where $m_i(b)$ is a continuous function with a maximum at $e^{R_i \hat{b}_i}$. Eq. 41 then generates

$$\frac{1}{t_i} = \frac{2e^{R_i \hat{b}_i} - 1}{\tau_i}, \qquad R_i \Phi_i = 2e^{R_i \hat{b}_i} - 1 \tag{44}$$

In order for the industry to grow ($\tau_i > t_i$), one must have that $R_i \Phi_i > 1$. This will occur if $R_i \hat{b}_i > 0$, which is always true if there is a period of positive reinvestment that occurs, in other words if more money is reinvested than the threshold necessary to just maintain the existing production capacity in operation (that threshold being $R_i = 0$). Whenever this is the case, the growth rate is larger than the death rate.

5. Discussion: the demographic phase in technology transitions

Starting from the picture of Geels (2002), the process of technology transitions could be thought of as going through two different phases. This is depicted in figure 3. New technologies originates from small, erratic, cumulative incremental innovations that gradually gain coordination as inventors and firms get to grips with understanding their own market and figuring out what is possible technically. This is shown with small randomly oriented arrows, with three colours indicating three innovations generating roughly the same service, or *societal function*. Many trials and errors generate experience and learning that gradually determine the successful direction to take. Once this happens, a better defined technology in a particular socio-technical context begins to gain momentum of diffusion, and enters what I will call the *demographic phase*. At this point, the growth rate is determined *both* by: (1) consumer preferences (in terms of the respective advantages and flaws of competing technologies) and related socio-technical context and its evolution, (2) the timescales of birth and death, or technology turnover. In a situation of very clear and favourable consumer preferences and socio-technical evolution, the diffusion becomes limited by the birth rate of the new technology, and by the death rate of the old technology being replaced. The birth rate cannot be faster than the rate of investment into production capital and labour, due to the magnitude of the income and associated financial flows, while the death rate cannot be faster than a certain lifetime associated to either the technical wearing out of units or to their *sunk costs*.

The innovation phase is difficult to model in a forecasting context, as this would require knowing the unknown, inventions that have not yet been invented. Therefore, it is difficult to describe innovation quantitatively beyond the picture by Geels (2002). However, when technologies enter the demographic phase, once they are well defined, modelling their evolution becomes straightforward, given a model of technology choice and knowledge of the birth and survival functions $\ell(a)$ and $m(b)$, or alternatively the life expectancy and the rate of reinvestment into production capital and labour of all competing technologies. Henceforth the applicability of the model becomes affected by either the possibility of new innovations appearing later into the picture, which cannot be predicted, or whether the conditions enumerated in section 4.1 remain met.

The key property that produce these limitations on the possible rates of growth and decline of technologies is that of *self-correlation*, where the number of units (or market shares) of a technology depends on itself in the past, the extent of which is defined by the functions $\ell(a)$ and $m(b)$. This self-correlation determines the overall rate of change of technology, the technology turnover. This rate of change is the key property to understand in contexts where technological change is important, notably in climate change mitigation. In climate change mitigation, the extent of climate change will be determined by future cumulative emissions, closely related to the moment when emissions will peak (if at all). Unless society is willing to accept or the economy able to afford a significant amount of early scrapping of technology (decommissioning technology much before its payback time), even in scenarios of strong policy incentives for change, this moment is critically determined by the rate of technology turnover, which is produced by this self-correlation.

6. Conclusion

This work has demonstrated that the origin of the empirical observation of the applicability of the Lotka-Volterra model of competition dynamics to technology diffusion originates from demographic principles applied to technology. I have created an age structured model of technology demography, using life expectancies and birth rates, and have created a complex model that, given the right conditions, using an approximation, falls back onto the form of the Lotka-Volterra model of competition. This operation has demonstrated the origin of the scaling parameters of the Lotka-Volterra model, the timescales of technology diffusion, in terms of *demographic* properties of technology, namely their birth rates and life expectancies.

The calculation presented however generates more insight than the simple correspondence of the Lotka-Volterra system to demography. While every previous quantitative use of the Lotka-Volterra for modelling technology diffusion has remained empirical and without clear justification, the calculation presented here explains *why* the Lotka-Volterra actually describes well systems of competing technologies at all. It moreover clarifies under which conditions it applies. Meanwhile, this paper gives meaning of the timescales of technology population dynamics suggested as a very general principle in the evolutionary economics literature and evolutionary game theory. The reasoning was broadened in order to connect to other descriptions of technology diffusion, in particular that of socio-technical systems and the multi-level perspective. I have defined a *demographic phase* of technology diffusion which, after technologies have emerged in protective niches, the seeds of diffusion, appropriate changes in the environment can enable to grow and invade the technology landscape.

This presentation clarifies the meaning of the scaling constants of the Lotka-Volterra model that enables its parameterisation without prior empirical measurement, difficult to do in cases where only small amounts of data is available. This tends to be the case precisely in the cases that are of most interest, namely when exploring the diffusion potential of new technologies under different assumptions concerning the market environment such as policy. This model enables to build models of technology based onto S-shaped diffusion curves and to parameterise them using known properties of the technologies and those of their respective production industries.

Finally, this work expands significantly the theoretical description of the approach used in the FTT family of technology models, including *FTT:Power*, which is meant to create a paradigm shift to conventional overall cost optimisation and the social planner assumption. As described in Mercure et al. (2013), the current standard calculation of greenhouse gas emissions using cost-optimisation models is a conceptually flawed concept, which could lead the climate change mitigation research and climate policy communities in error. This opens up a new avenue for quantitative technology modelling which could generate insight in many fields of research.

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